Truth Tables for Conditionals and Biconditionals

1 Truth Table for Conditionals

To make sense of the possibilities for a Conditional it will be useful to think of a statement that could in effect be a contract between two people. I like to use this one:

"I told my son, If you do well on your math test Then I will buy you an ice cream"

Changing this into symbolic form we get:

\[ p \rightarrow q \]

So there are four possibilities for this two statement sentence:

1) My son does well and I buy him ice cream (I kept the contract = T )
2) My son does well and I do not buy him ice cream (I did not keep the contract = F )
3) My son doesn't do well and I buy him ice cream (I spoiled my son, but did not break my contract, = T )
4) My son doesn't do well and I do not buy him ice cream (I kept the contract = T)

The Truth Table will look like this:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \rightarrow q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

In English this would be..."Only False when first statement is True and second is False, TF=F"

Example 1 Construct a Truth Table for \( \sim q \rightarrow p \)

Look for the TF line and give the answer below the \( \rightarrow \) as F. All of the others must be True.

\[
\begin{array}{c|c|c}
\text{p} & \text{q} & \sim p \rightarrow q \\
T & T & F T T \\
T & F & F T F \\
F & T & T T T \\
F & F & T F F \\
\end{array}
\]

Thus the answer will be: \( \sim q \rightarrow p = T T T F \)
Example 2  Construct a Truth Table for \((-p \lor q) \rightarrow (p \land \neg q)\)

Start by filling in both known values for each grouped statement:

\[
\begin{array}{c|c|c|c|c}
 p & q & (-p \lor q) & (p \land \neg q) \\
 T & T & F & T \\
 T & F & F & T \\
 F & T & T & F \\
 F & F & T & T \\
\end{array}
\]

For the first group we have an OR statement, so we search for the FF row, and give it an \(\neg F\). All the others will be True.
In the second group, we have a AND statement, so we search for the TT row and give this a \(T\). All the others will be False.

\[
\begin{array}{c|c|c|c|c|c|c}
 p & q & (-p \lor q) & (p \land \neg q) \\
 T & T & F & T & T & F & T \\
 T & F & F & F & T & T & T \\
 F & T & T & F & F & T & T \\
 F & F & T & F & F & T & T \\
\end{array}
\]

Now of course you would complete this in one table and note what the solution would be. But the above is a step by step of the operations that you would complete and which two columns combine to the answer.

The answer for: \((-p \lor q) \rightarrow (p \land \neg q)\) is \(FTFF\)

2  Truth Table for Biconditional

By definition a Biconditional is Two Conditionals, just in opposite directions:

\[p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)\]

Thus the name Biconditional.

So to find the Base Table for Biconditionals we only need to find the answer for \((p \rightarrow q) \land (q \rightarrow p)\)

\[
\begin{array}{c|c|c|c|c|c|c|c}
 p & q & (p \rightarrow q) & (q \rightarrow p) \\
 T & T & T & T & T & T & T & T \\
 T & F & F & F & T & F & F & T \\
 F & T & T & F & T & F & F & T \\
 F & F & F & T & T & F & F & T \\
\end{array}
\]

then finding the AND between the Green we find

\[
\begin{array}{c|c|c|c|c|c|c|c}
 p & q & (p \rightarrow q) & (q \rightarrow p) \\
 T & T & T & T & T & T & T & T \\
 T & F & F & F & T & F & F & T \\
 F & T & T & F & T & F & F & T \\
 F & F & F & T & T & F & F & T \\
\end{array}
\]

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Thus the Truth Table for a Biconditionals is:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ↔ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

In English, "**Only True when both are the Same**" I think of it as (+) (+) = (+) and (−) (−) = (+)

**Example 3** Construct a Truth Table for \( \sim (p \rightarrow r) \lor (\sim q \leftrightarrow r) \)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>( \sim (p \rightarrow r) \lor (\sim q \leftrightarrow r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Thus the solution to \( \sim (p \rightarrow r) \lor (\sim q \leftrightarrow r) \) is **TTTFTTF**

**Example 4** Determine the Truth Value of \( \sim (p \rightarrow q) \land [(\sim r \lor q) \leftrightarrow \sim p] \)

Given that \( p = T \), \( q = T \), and \( r = F \)

\[
\sim (p \rightarrow q) \land [(\sim r \lor q) \leftrightarrow \sim p] \\
\sim ((p \rightarrow q) \land [(\sim r \lor q) \leftrightarrow \sim p]) \\
\sim ((T \rightarrow F) \land [(\sim F \lor T) \leftrightarrow \sim (F)]) \\
\sim (T \rightarrow F) \land [(T \lor T) \leftrightarrow F] \\
\sim F \land [T \leftrightarrow F] \\
T \land F \\
F
\]

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**Example 5** Determine the Truth Value of "If $1 + 2 = 4$ then $2 - 5 = 3$"

\[
\begin{array}{c|c}
\text{p} & 1 + 2 = 4 \quad (\text{false}) \\
\text{q} & 2 - 5 = 3 \quad (\text{false})
\end{array}
\]

In symbolic form this statement will be $p \rightarrow q$ and thus $F \rightarrow F = T$

So even though both simple statements are False, it is still a True statement!

**A few Side notes:**

**Definitions:**

1. Self-Contradiction - A compound statement that is always False (truth table answers all F)
2. Tautology - A compound statement that is always True
3. Implication - A Conditional that is a Tautology (always true)