Installment Buying  (part A, Fixed Loans)

When we get a car loan, a mortgage, a personal loan, or even use a credit card, we are purchasing something under "credit". We sign agreements on how we are going to pay back this money, and oftentimes it is an agreement to pay back a portion of the debt with monthly installments (monthly payments).

Some definitions that will help us discuss portions of the ideas behind "Installment Buying" that we will use:

**Open Ended Credit** - This is Credit Cards. You can borrow $100 today and then $50 more tomorrow, the balance is always fluctuating.

**Fixed Loan** - This is a car loan, a mortgage, or a personal loan. You borrow money and have to pay back in a fixed time. Fixed amount and fixed time limit.

**Finance Charge** - The amount of money paid, to borrow money. The money paid above and beyond the repayment of the principal.

**Total Installment** - The total money paid back to the lending institution. Principal + Finance Charge.

To find the amount of money needed to pay back each month for a Fixed Loan, we will use the table below. This table is the Finance Charge owed given a certain time period and interest rate. It is based on a fixed principal of $100.

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<tr>
<th>Time (months)</th>
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1  Using the Table.

If we borrow $100 for 48 months at 9.5%, then the finance charge will be $20.59. If we borrow $100 for 30 months at a rate of 12.5%, then the finance charge will be $16.95.
In the case of the 30 month loan above, if we borrowed $250 at 12.5% then we would just take the Finance Charge and multiply it by 2.5. To find Finance Charge we could design a formula:

\[ FC = \frac{PV}{100} \]

where \( P \) is principal, \( V \) is the table value for a certain number of months at a specific interest rate, and the division by 100 will tell us how many $100 that we are borrowing.

## 2 Finding the Monthly Payment

**Example 1** George sees an add for a couch that costs $800. The store says it will sell the couch for no money down and 12.5% for 2 years. What will be the Finance Charge? What will be the monthly payment on this loan?

Use:

\[ FC = \frac{(Principal \times Table \ Value)}{100} \]

\[ FC = \frac{(800 \times 13.54)}{100} \]

\[ FC = 108.32 \]

So it will cost an extra $103.84 to finance the couch. To find the monthly payments we only need to divide the Total Installment into equal payments over 2 years

\[ TI = P + FC \]

\[ TI = 800 + 108.32 \]

\[ TI = 908.32 \]

\[ Payment = \frac{908.32}{24} \]

\[ Payment = 37.85 \text{/month} \]

## 3 Finding the Interest Rate

What if we forgot the interest rate that we have for a loan? We would know the monthly payments. We would know how much the principal was (usually). We would know how long we need to pay the loan. Let's see...

**Example 2** Mary bought a car for $8500 in 1988 and pays $221.76 per month for 4 years. What interest rate did she get for this loan?

This is just going backwards from the above problem. First let us find the Total Installment:

\[ TI = (monthly \ payment \times number \ of \ months) \]

\[ = (221.76 \times 48) \]

\[ = 10644.48 \]
Then we need to find the Finance Charge:

\[ TI = P + FC \]

\[ 10644.48 = 8500 + FC \]

\[ FC = 10644.48 - 8500 \]

\[ FC = \$2144.48 \]

Then we need the Table Value:

\[ FC = \frac{\text{(Principal) (Table Value)}}{100} \]

\[ 2144.48 = \frac{(8500) (Table Value)}{100} \]

\[ (2144.48) (100) = (8500) (TV) \]

\[ TV = \frac{(2144.48) (100)}{8500} \]

\[ TV = 25.23 \]

We need to look up on the table, for 48 months the interest that gives a FC of \$25.23. Moving Left to Right over the 48 months values we need the closest value to this 25.23.

**Finance Charge per \$100 borrowed**

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Thus the interest RATE is 11.5%.

**NOTE:** For all of these problems, by the end, you should have been given or have found:

(a) Finance Charge  
(b) Total Installment  
(c) Table Value  
(d) Monthly Payment  
(e) Principal  
(f) Number of Months  
(g) Interest Rate
4 Paying Loan Early

What if we wanted to pay a loan off early? Would you take your monthly payment and multiply by how many payments are left? NO. When you pay a loan off early, you will save on the interest that is already computed for your monthly payments. We are reducing the amount of time that the interest was calculated from.

To find the amount of money that can be saved, I will use the Actuarial Method and not the Rule of 78’s (which is just an approximation of how much is saved)

The money saved is given by the formula:

\[ u = \frac{kRV}{100 + V} \]

where:

- \( u \) is unearned interest (the amount of money that you will save)
- \( k \) is the number of payments that you are paying off early (not the one scheduled payment)
- \( R \) is the Required monthly payment
- \( V \) is the Table Value associated with \( k \) and the interest rate of the loan.

Example 3 Carrie bought her SUV for $23750. The dealership got her a loan that paid 90% of the purchase price (10% down payment) of the car at 7% for 5 years.

Part A- Find the Monthly Payment for this car (R)

1. (a) What is the principal on her loan?
   (b) What is her Finance Charge?
   (c) How much does she pay per month?

1. (a)

\[ P = .90 \times 23750 \]
\[ = 21375 \]

(b)

\[ FC = \frac{(\text{Principal})(\text{TV}=7\% \text{ 60 months})}{100} \]
\[ = \frac{(21375)(18.81)}{100} \]
\[ = 4020.64 \]
Part B - Find the savings by paying off early (u)

Example 4  What if she wins the lottery and wants to pay off her loan on her 24th payment (this will include her 24th payment and 60-24 more payments (60 – 24 = 36). She will have to save money on the 24 payments that she will be making "early". The formula is:

\[ u = \frac{k \cdot R \cdot V}{100 + V} \]

where \( k \) is the number of months that are being paid off early, \( R \) is the Monthly Payment, \( V \) is the table Value associated with \( k \) and the interest rate of the loan.

All we need for this formula is the \( V \) from the table. Use \( k = 36 \) months and 7% interest and the table gives us: \( V = 11.16 \)

\[
\begin{align*}
  u &= \frac{(36) \cdot (423.26) \cdot (11.16)}{100 + 11.16} \\
  &= \frac{170049.}{111.16} \\
  &= $1529.77
\end{align*}
\]

So she will save $1529.77 by paying the loan off 36 months early. To find out how much she will owe the bank with her 24th payment, we need to find out how much she would pay for 37 payment coupons (the 24th payment plus the 36 she is paying early). From this we will take away the money she is saving

\[
(423.26) (37) - 1529.77 \\
= $14130.90
\]

So she will have to cut a check for $14,130.90 to finally own her SUV outright.