Equivalent Statements,and DeMorgan's Law

1 Equivalent Statements ($\Leftrightarrow$ or $\equiv$)

Two (or more) statements that have the same Truth Table Answers.

Example 1 *Is* $p \wedge (q \vee \sim p)$ *equivalent to* $(p \wedge q) \vee (p \wedge \sim p)$ *?*

The best way to find out whether these statements are equivalent will be to find each truth table solution:

p	*q*	*p*	∧	*(q*	∨	*~p)*
T	*T*	*T*	***T***	*t*	*T*	*f*
T	*F*	*T*	***F***	*f*	*F*	*f*
F	*T*	*F*	***F***	*t*	*T*	*t*
F	*F*	*F*	***F***	*f*	*T*	*t*

$\Leftrightarrow$

p	*q*	*(p*	∧	*q)*	∨	*(p*	∧	*~p)*
T	*T*	*t*	*T*	*t*	***T***	*t*	*F*	*f*
T	*F*	*t*	*F*	*f*	***F***	*t*	*F*	*f*
F	*T*	*f*	*F*	*t*	***F***	*f*	*F*	*t*
F	*F*	*f*	*F*	*f*	***F***	*f*	*F*	*t*

Since these truth tables have the exact same solution, they are deemed Equivalent. Saying one is identical as saying the other! Lawyers love this stuff!!

2 Demorgan's Law

$$\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$$
$$\sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$$

Proof of first statement

By completing each truth table we find that they both have the same solution, thus they are equivalent.

p	*q*	~	*(p*	∨	*q)*
T	T	**F**	t	T	t
T	F	**F**	t	T	f
F	T	**F**	f	T	t
F	F	**T**	f	F	f

$\Leftrightarrow$

p	*q*	*~p*	∧	*~q*
T	T	f	**F**	f
T	F	f	**F**	t
F	T	t	**F**	f
F	F	t	**T**	t

The proof of the second statement is very similar.

2.1 Using Demorgan's Law

Demorgan's offers a quick way of rewriting some negated compound statements. Notice that the rule of Demorgan's, one side has no () and the other side does. Also the () has a negation sitting in front of it. Thus when an OR or AND statement does not have () then under Demorgan's, its equivalent statement will.

Example 2 *Use Demorgan's to rewrite the given statement* $\sim p \vee q$

First notice that there is not a set of () . *Therefore a set will be needed, with a Negation in front. For the "inside" make the* $\sim p$ *became* p, *the* $\vee$ *became* $\wedge$, *and the* q *became a* $\sim q$.

$$\begin{aligned} \sim p \vee q &\Leftrightarrow \sim () \\ &\Leftrightarrow \sim (p \wedge \sim q) \end{aligned}$$

Example 3 *Demorgan's is also useful in changing English Statements:*

Use Demorgan's Law to rewrite as an equivalent statement:

" There is a snake in my boot or I am not smiling"

Changing into symbolic form and using Demorgan's Law...

p: *There* ***is*** *a snake in my boot*
q: *I* ***am*** *smiling*

$$p \vee \sim q \Leftrightarrow \sim (\sim p \wedge q)$$

And finally write out in English...

"It is not true that there is not a snake in my boot and I am smiling"

Example 4 *Use Demorgan's Law to rewrite as an equivalent statement:*

"It is false that the Earth is not round or the Sun orbits around the Earth"

Changing into symbolic form and using Demorgan's Law...

p: *The Earth is round*
q: *The Sun orbits the Earth*

$$\sim (\sim p \vee q) \Leftrightarrow p \wedge \sim q$$

And finally write this out in English...

" The Earth is round and the Sun does not orbit around the Earth"

2.2 A strange Equivalence:

Not much used, but texts always use:

$$p \rightarrow q \Longleftrightarrow \sim p \vee q$$

Lets give this some simple statements so we can marvel at the strangeness of this equivalence.

(prove on your own that this is truly an equivalence relation by using a Truth Table!!)

Example 5 *Use* $p \rightarrow q \Longleftrightarrow \sim p \vee q$ *to rewrite statement in another equivalent form...*

" If George is riding his bike then Mary is not sleeping in the shade"

p: *George is riding his bike*
q: *Mary is sleeping in the shade* $p \rightarrow \sim q \Leftrightarrow \sim p \vee \sim q$

Notice that the q *in the "Law" does not change, just the* p*…*

Thus the statement would become:

" George is not riding his bike OR Mary is not sleeping in the shade."

3 Conditional Equivalences

A major portion of Equivalences involve the Conditional Statement ($p \rightarrow q$)

Conditional $p \rightarrow q$

p	q	p	$\rightarrow$	q
T	T	t	T	t
T	F	t	F	f
F	T	f	T	t
F	F	f	T	f

Converse $q \rightarrow p$

p	q	q	$\rightarrow$	p
T	T	t	T	t
T	F	f	T	t
F	T	t	F	f
F	F	f	T	f

Inverse $\sim p \rightarrow \sim q$

p	q	$\sim p$	$\rightarrow$	$\sim q$
T	T	f	T	f
T	F	f	T	t
F	T	t	F	f
F	F	t	T	t

Contrapositive $\sim q \rightarrow \sim p$

p	q	$\sim q$	$\rightarrow$	$\sim p$
T	T	f	T	f
T	F	t	F	f
F	T	f	T	t
F	F	t	T	t

Notice that the Conditional is equivalent to the Contrapositive and that the Converse is equivalent to the Inverse.

You can test this out with the simple statements:

p: A number is Divisible by 4
q: A number is Divisible by 2

"If a number is divisible by 4 then it is divisible by 2" is the conditional

and " If a number is not divisible by 2 then it is not divisible by 4" is the contrapositive, where both are True Statements. Try the other two, Converse and Inverse, using these simple statements, and those statements should both be False.

Example 6 *Find the Converse, Inverse, and Contrapositive of the following Statement:*

"If the bank was robbed then I will not have any money"

p: *The bank was robbed*
q: *I have money* $p \rightarrow \sim q$

I would highly suggest a table format to this problem:

Conditional	$p \rightarrow \sim q$
	"If the bank was robbed then I will not have any money"
Converse (flip)	$\sim q \rightarrow p$
	"If I do not have any money then the bank was robbed "
Inverse (change)	$\sim p \rightarrow q$
	"If the bank was not robbed then I have money"
Contrapositive (flip & Change)	$q \rightarrow \sim p$
	"If I have any money them the bank was not robbed "